Econ 802

Second Midterm

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All questions have equal weight. If anything is unclear, please ask.

1. The elasticity of scale is closely related to the shape of the cost curves.

(a) Let x* be the cost-minimizing input bundle for the output y*. Prove that e(x*) = AC(y*)/MC(y*) where e(x) is the elasticity of scale, AC is long run average cost, and MC is long run marginal cost. Hint: use the fact that MC equals the Lagrange multiplier in the cost minimization problem.

(b) Use the result in (a) to give a short proof of the fact that a Cobb-Douglas production function cannot yield a U-shaped long run average cost curve. Explain graphically.

2. At prices $p^A = (1,1)$, Joe Consumer chooses the bundle $x^A = (0,6)$. At prices $p^B = (1,4)$, Joe chooses the bundle $x^B = (4,2)$.

(a) Draw a graph showing the budget lines and the chosen bundles in these two cases. Assuming Joe's preferences are locally non-satiated, what is the largest possible upper contour set of the form $\{x \geq 0: u(x) \geq u(x^A)\}$? Show this set on your graph and give a careful verbal explanation.

(b) Assume Joe's utility function has the form $u(x) = ax_1 + bx_2 + c$. Using information already provided, what restrictions can you impose on the values of the parameters a, b, and c? Do you know enough to compute Joe's Marshallian demand functions? Why or why not? (Note: don't try to calculate these functions, I just want to know whether you could do it in principle.)

3. Suppose Jane's Marshallian demand functions are $x_1 = p_2/p_1$ and $x_2 = (m/p_2) - 1$.

(a) On one graph show Jane's income expansion path, and on another graph show her price offer curve as p_1 changes. Briefly justify your answer in each case.

(b) Without using the indirect utility function or the expenditure function, show that

$$\partial h_2[p,v(p,m)]/\partial p_1 = \partial h_1[p,v(p,m)]/\partial p_2$$

where $h_i(p,u)$ is the Hicksian demand function for good i.

4. Consider the indirect utility function $v(p,m) = \ln (m/2) - (\ln p_1 + \ln p_2)/2$.

(a) What is the direct utility function u(x)?

(b) Suppose there are i = 1 . . n consumers, each with indirect utility function $v(p,m_i) = \ln (m_i/2) - (\ln p_1 + \ln p_2)/2$ where m_i is consumer i's income. Derive the aggregate demand $X^k(p_1,p_2,m_1 \ldots m_n)$ for good k = 1,2. Do these aggregate demands depend on how income is distributed among the consumers? Why or why not?